

Anisotropic universe space-time non-commutativity and scalar particle creation in the presence of a constant electric field

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Abstract

We study the effect of the non-commutativity on the creation of scalar particles from vacuum in the anisotropic universe space-time. We derive the deformed Klein-Gordon equation up to second order in the non-commutativity parameter using the general modified field equation. Then the canonical method based on Bogoliubov transformation is applied to calculate the probability of particle creation in vacuum and the corresponding number density in the k mode. We deduce that the non-commutative space-time introduces a new source of particle creation.

KEYWORDS: Non-commutative field theory, Bogoliubov transformation, Particle production.

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1 Introduction

In the classical theory black holes can only absorb and not emit particles. However it was shown that quantum mechanical effects cause black holes to create and emit particles. It is also well known that the most significant prediction of this theory is the phenomenon of particle creation which leads to the concept of quantum gravity. In this paper we are interested in the issue of particle production by constructing a simple type of the non-commutative geometry.

The extension of quantum field theory to one in a curved space is the starting point towards quantum gravity in curved space-time. However another important concept in the context of quantum gravity is non-commutative geometry, by which the quantization of the space-time leads to quantifying gravity. Thus the non-commutative space-time is intrinsically connected to quantum gravity despite the well-known problem of Lorentz-violating symmetry. All other fundamental problems, such as the unitarity violation [4], causality [5] and UV/IR divergences [6], have been discussed in the context of the local Lorentz invariance.

In ref. [7], the authors showed that these problem can arise by inducing a non-constant metric into the theory and they found that at high energy gravity and non-commutative geometry must become dependant on each other. Several important works were performed in a formally Lorentz-invariant approach (see for example the reviews [2, 8, 9]). Various theories of gravity in the context of non-commutative geometry have amongst others been studied in refs. [10 – 22] and cosmology on non-commutative space-time has been explored in ref. [23]. Even certain ideas referring to quantum gravity have been explored with respect to non-commutative geometry, see refs. [24 – 28] for example. Another approach is based on the twisted Poincaré algebra constructed for canonically deformed space with a constant parameter of non-commutativity, where in this formalism the Lagrangian density is invariant and the gauge and pure gravity theories are consistent.

In our previous work [29] we have attempted to construct a non-commutative gauge gravity model, where the problem of the unitarity (see for example refs. [4, , 14, 15, 30, 31, 32]) is overcome by the construction of generalized local Lorentz and general coordinate transformations, which preserve the non-commutative coordinate canonical commutation relations. The phenomenon of scalar particle creation in anisotropic Bianchi I universe with a constant electric field has been analyzed in ref. [33]. Actually, there is no electric charge in the universe to create an electric field, meaning that the particles can be created from vacuum by the expansion of the universe itself with no other external field present.

The aim of this paper is the study the effect of the non-commutativity on the creation of scalar particles from vacuum in the space-time anisotropic Bianchi I universe when a constant electric field is present. We compute the number density of created particles in the cases of strong and weak field. From our results we clearly deduce that the the non-commutativity plays the role of the electric field.

This paper is organized as follows. In section 2 we derive the correspond-

ing Seiberg-Witten maps up to the first order of θ for the various dynamical fields and we propose an invariant action of the pure non-commutative gauge gravity and non-commutative charged scalar field in interaction. In section 3 we derive the anisotropic universe non-commutativity space-time Klein-Gordon (KG) equation and obtain its solution. Then we compute the density of created scalar particles and discuss the weak and strong field limits. The last section is devoted to a discussion.

2 Seiberg-Witten maps

One can get at first order in the non-commutative parameter $\theta^{\mu\nu}$ the following Seiberg-Witten maps [1]:

$$\begin{aligned}
\hat{\varphi} &= \varphi - \frac{1}{2}\theta^{\mu\nu}A_\nu\partial_\mu\varphi + \mathcal{O}(\theta^2), \\
\hat{\lambda}_P &= \lambda_P + \frac{1}{4}\theta^{\sigma\rho}\{\partial_\sigma\lambda_P, \omega_\rho\} + \mathcal{O}(\theta^2), \\
\hat{\lambda}_G &= \lambda_G + \frac{1}{4}\theta^{\sigma\rho}\{\partial_\sigma\lambda_G, A_\rho\} + \mathcal{O}(\theta^2), \\
\hat{A}_\xi &= A_\xi - \frac{1}{4}\theta^{\mu\nu}\{A_\nu, \partial_\mu A_\xi + F_{\mu\xi}\} + \mathcal{O}(\theta^2), \\
F_{\mu\xi}^1 &= \frac{1}{2}\theta^{\alpha\beta}\{F_{\mu\alpha}F_{\xi\beta}\} - \frac{1}{4}\theta^{\alpha\beta}\{A_\alpha, (\partial_\beta + D_\beta)F_{\mu\xi}\} + \mathcal{O}(\theta^2), \\
\hat{e}_\mu^a &= e_\mu^a - \frac{i}{4}\theta^{\alpha\beta}(\omega_\alpha^{ac}\partial_\beta e_\mu^c + (\partial_\beta\omega_\mu^{ac} + R_{\beta\mu}^{ac})e_\mu^c) + \mathcal{O}(\theta^2),
\end{aligned} \tag{1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu], \tag{2}$$

$$\omega_\mu = \omega_\mu^{ab}S_{ab}, \tag{3}$$

$$\hat{A}_\mu = \hat{A}_\mu^a T^a = \hat{A}_\mu^a * \hat{e}_\mu^k T^a, \tag{4}$$

$$\hat{\omega}_\mu = \hat{\omega}_\mu^{ab}S_{ab} = \hat{\omega}_\mu^{ab} * \hat{e}_\mu^k S_{ab}, \tag{5}$$

$$\theta^{\mu\nu} = \hat{e}_{*a}^\mu * \hat{e}_{*b}^\nu \theta^{ab}, \tag{6}$$

and ω_μ^{ab} are the spin connections and \hat{e}_{*a}^μ is the inverse-* of the vierbein \hat{e}_μ^a defined as:

$$\hat{e}_\mu^b * \hat{e}_{*a}^\mu = \delta_a^b, \tag{7}$$

and

$$\hat{e}_\mu^a * \hat{e}_{*a}^\nu = \delta_\mu^\nu. \tag{8}$$

To begin we consider a non-commutative gauge theory with a charged scalar particle in the presence of an electrodynamic gauge field in a general curvilinear system of coordinates. We can write the action as:

$$\mathcal{S} = \frac{1}{2\kappa^2} \int d^4x (\mathcal{L}_G + \mathcal{L}_{SC}), \tag{9}$$

where \mathcal{L}_G and \mathcal{L}_{SC} stand for the pure gravity and matter scalar densities corresponding to the charged scalar particle in the presence of an electric field, and where

$$\mathcal{L}_G = \hat{e} * \hat{R}, \quad (10)$$

and

$$\mathcal{L}_{SC} = \hat{e} * \left(\hat{g}^{\mu\nu} * \left(\hat{D}_\mu \hat{\varphi} \right)^\dagger * \hat{D}_\nu \hat{\varphi} + m^2 \hat{\varphi}^\dagger * \hat{\varphi} \right). \quad (11)$$

The deformed tetrad and scalar curvature are given by:

$$\hat{e} = \det_*(\hat{e}_\mu^a) \equiv \frac{1}{4!} \epsilon^{\mu\nu\rho\sigma} \varepsilon_{abcd} \hat{e}_\mu^a * \hat{e}_\nu^b * \hat{e}_\rho^c * \hat{e}_\sigma^d, \quad (12)$$

$$\hat{R} = \hat{e}_{*a}^\mu * \hat{e}_{*b}^\nu * \hat{R}_{\mu\nu}^{ab}, \quad (13)$$

and the gauge covariant derivative is defined as: $\hat{D}_\mu \hat{\varphi} = (\partial_\mu - ie\hat{A}_\mu) * \hat{\varphi}$.

In the following we consider a symmetric metric $\hat{g}_{\mu\nu}$ such that:

$$\hat{g}_{\mu\nu} = \frac{1}{2} (\hat{e}_\mu^b * \hat{e}_{\nu b} + \hat{e}_\nu^b * \hat{e}_{\mu b}). \quad (14)$$

As a consequence, the first-order expansion in the non-commutative parameter $\theta^{\alpha\beta}$ of the scalar curvature \hat{R} and metric $\hat{g}_{\mu\nu}$ vanishes. Thus \hat{R} and $\hat{g}_{\mu\nu}$ can be rewritten as:

$$\hat{R} = R + \mathcal{O}(\theta^2), \quad (15)$$

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + \mathcal{O}(\theta^2), \quad (16)$$

Next we use the generic field infinitesimal transformations ($\delta_{\hat{\lambda}} \hat{\varphi} = i\hat{\lambda} * \hat{\varphi}$), and the star-product tensor relations. We can prove that the action in eq. (34) is actually invariant. By varying the scalar density under the gauge transformation and from the generalised field equation and the Noether theorem we obtain [10]:

$$\frac{\partial \mathcal{L}}{\partial \hat{\varphi}} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \hat{\varphi})} + \partial_\mu \partial_\nu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \partial_\nu \hat{\varphi})} + \mathcal{O}(\theta^2) = 0. \quad (17)$$

3 The solution to the non-commutative Klein-Gordon equation and particle creation process

In this section we examine the particle creation phenomenon induced by vacuum instabilities in the context of the non-commutative geometry in presence the external vector potential A_μ . We shall take the example of the Klein-Gordon equation in a cosmological anisotropic non-commutative Bianchi *I* universe.

The deformed line element of the Bianchi *I* universe up to the first-order of θ takes the following form:

$$ds^2 = -dt^2 + t^2 (dx^2 + dy^2) + dz^2 + g_{\mu\nu}^{(1)} dx^\mu dx^\nu + \mathcal{O}(\theta^2). \quad (18)$$

We choose for $\theta^{\alpha\beta}$ the following form:

$$\theta^{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & \theta \\ 0 & 0 & \theta & 0 \\ 0 & -\theta & 0 & 0 \\ -\theta & 0 & 0 & 0 \end{pmatrix}, \quad \alpha, \beta = 0, 1, 2, 3. \quad (19)$$

We follow the same steps outlined in ref. [33] and look for the non-commutative correction of the metric up to the first order in θ .

Choosing the following diagonal tetrads:

$$\underline{e}_\mu^0 = (1, 0, 0, 0), \quad (20)$$

$$\underline{e}_\mu^1 = (0, t, 0, 0), \quad (21)$$

$$\underline{e}_\mu^2 = (0, 0, t, 0), \quad (22)$$

$$\underline{e}_\mu^3 = (0, 0, 0, 1), \quad (23)$$

then the nonzero spin connections are

$$\omega_1^{01} = -\omega_1^{10} = 1, \quad (24)$$

$$\omega_2^{02} = -\omega_2^{20} = 1. \quad (25)$$

Using the Seiberg-Witten map (1) and the choice (19) we can obtain the following deformed vierbeins:

$$\hat{\underline{e}}_\mu^0 = (1, 0, 0, 0), \quad (26)$$

$$\hat{\underline{e}}_\mu^1 = (0, t, -i\frac{\theta}{4}t, 0), \quad (27)$$

$$\hat{\underline{e}}_\mu^2 = (0, i\frac{\theta}{4}t, t, 0), \quad (28)$$

$$\hat{\underline{e}}_\mu^3 = (0, 0, 0, 1). \quad (29)$$

As a consequence, the first-order expansion in the non-commutative parameter $\theta^{\alpha\beta}$ of the Bianchi I metric vanishes. Thus (18) can be rewritten as:

$$ds^2 = -dt^2 + t^2 (dx^2 + dy^2) + dz^2 + \mathcal{O}(\theta^2). \quad (30)$$

In order to identify the particle states we follow the quasi-classical approach of ref. [34]. The standard method is to specify the positive and negative frequency modes and solve the classical Hamilton-Jacobi equation looking specifically for the asymptotic limits of the solution $t \rightarrow 0$ and $t \rightarrow \infty$. Then one solves the Klein Gordon equation by comparing with the quasi-classical limits, and specifying the positive and negative frequency states. Finally one utilises Bogoliubov transformations and calculates the number density for created particles.

Using the modified field equation (17) with the generic boson field $\hat{\varphi}$ one can find in a non-commutative curved space-time and in the presence of the external potential \hat{A}_μ the following modified Klein-Gordon equation:

$$(\eta^{\mu\nu} \partial_\mu \partial_\nu - m_e^2) \hat{\varphi} + \left(i e \eta^{\mu\nu} \partial_\mu \hat{A}_\nu - e^2 \eta^{\mu\nu} \hat{A}_\mu * \hat{A}_\nu + 2 i e \eta^{\mu\nu} \hat{A}_\mu \partial_\nu \right) \hat{\varphi} = 0, \quad (31)$$

with the deformed external potential $\hat{A}_\mu = (0, 0, 0, Et)$ in free non-commutative space-time being:

$$\hat{a}_3 = a_3 - \Theta^{\mu k} a_k \partial_\mu a_3 + \mathcal{O}(\Theta^2). \quad (32)$$

For a non-commutative time-space we have $\Theta^{03} \neq 0$ and $\Theta^{ki} = 0$, where $i, k = 1, 2, 3$. In this case we can write:

$$\eta^{\mu\nu} \partial_\mu \partial_\nu = -\partial_0^2 + \frac{1}{t^2} (\partial_1^2 + \partial_2^2) + \partial_3^2, \quad (33)$$

and

$$2ie\eta^{\mu\nu} \hat{A}_\mu \partial_\nu = 2ieEt(1 + \theta E) \partial_3, \quad (34)$$

and

$$-e^2 \eta^{\mu\nu} \hat{A}_\mu * \hat{A}_\nu = [ieEt(1 + \theta E)]^2. \quad (35)$$

The Klein-Gordon equation (31) (in the presence of a constant external field A_μ) up to $\mathcal{O}(\theta^2)$ then simplifies to:

$$\left[-\partial_0^2 + \frac{1}{t^2} (\partial_1^2 + \partial_2^2) + \partial_3^2 - m^2 + 2ieEt(1 + \theta E) \partial_3 + [ieEt(1 + \theta E)]^2 \right] \hat{\varphi} = 0. \quad (36)$$

In order to keep our results compact and transparent we make use of the approximation:

$$1 + \theta g \approx \exp(\theta g), \quad (37)$$

with g being an arbitrary regular function. Equation (36) commutes with the operator $-i\vec{\nabla}$, and therefore the wave functions $\hat{\varphi}$ can be cast into:

$$\hat{\varphi} = \tilde{\Delta}(t) \exp(ik_x x + ik_y y + ik_z z). \quad (38)$$

Substituting eq. (38) into eq.(36), one can get:

$$\left[\frac{d^2}{dt^2} + \frac{k_\perp^2}{t^2} + k_z^2 + m^2 + 2e\tilde{E}tk_z + e^2\tilde{E}^2t^2 \right] \tilde{\Delta}(t) = 0, \quad (39)$$

where

$$\tilde{E} = E \exp(\theta E), \quad (40)$$

and the eigenvalue k_\perp is given by:

$$k_\perp = \sqrt{k_x^2 + k_y^2}. \quad (41)$$

We adopt the following change of variable:

$$\rho = ie\tilde{E}t^2, \quad (42)$$

and we deduce that for $k_z = 0$, equation (39) becomes:

$$\left[\frac{d^2}{d\rho^2} + \frac{1}{2\rho} \frac{d}{d\rho} + \frac{k_\perp^2}{4\rho} - \frac{1}{4} - i \frac{m^2}{4e\tilde{E}} \right] \tilde{\Delta}(\rho) = 0. \quad (43)$$

Following ref. [34], the solution to eq. (43) can be written as a combination of Whittaker functions $M_{\tilde{k}_\theta, \mu}(\rho)$ and $W_{\tilde{k}_\theta, \mu}(\rho)$:

$$\tilde{\Delta}(\rho) = \rho^{-1/4} \left(C_1 M_{\tilde{k}_\theta, \mu}(z) + C_2 W_{\tilde{k}_\theta, \mu}(z) \right), \quad (44)$$

where \tilde{k}_θ and μ are given by:

$$\tilde{k}_\theta = -i \frac{m^2}{4eE} \exp(-\theta E), \quad \mu = \frac{i}{2} \sqrt{k_\perp^2 - \frac{1}{4}}. \quad (45)$$

Then the general solution of (43) can be expressed in terms of the hypergeometric functions $F\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right)$ and $G\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right)$ as follows:

$$\begin{aligned} \tilde{\Delta}(\rho) = & C_1 \rho^{\mu+1/4} e^{-\rho/2} F\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right) + \\ & + C_2 \rho^{\mu+1/4} e^{-\rho/2} G\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right), \end{aligned} \quad (46)$$

where C_1 and C_2 are normalisation constants.

To construct the positive and negative frequency modes we use the asymptotic limit of the solution (46) and compare the result with that obtained by solving the Hamilton-Jacobi relativistic equation at $t = 0$ ($\rho = 0$). Thus it may be shown that the positive and negative frequency modes are given by:

$$\tilde{\Delta}_0^+ = C_0^+ \rho^{\mu+1/4} e^{-\rho/2} F\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right), \quad (47)$$

and

$$\tilde{\Delta}_0^- = \left(\tilde{\Delta}_0^+\right)^* = C_0^+ (-1)^{\mu+1/4} \rho^{\mu+1/4} e^{-\rho/2} F\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right), \quad (48)$$

where C_0^+ is normalisation constant. We note that the hypergeometric functions $F\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right)$ and $G\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right)$ have the following asymptotic limits:

$$F\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right) \sim 1 \quad \text{for} \quad |\rho| \ll 1, \quad (49)$$

$$G\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right) \sim \rho^{\tilde{k}_\theta - \mu - 1/2} \quad \text{for} \quad |\rho| \rightarrow \infty. \quad (50)$$

One can show that the positive and negative frequency modes for $|\rho| \rightarrow \infty$, by observing the asymptotic limit of $G\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right)$, is given by:

$$\tilde{\Delta}_\infty^+ = C_\infty^+ \rho^{\mu+1/4} e^{-\rho/2} G\left(\frac{1}{2} - \tilde{k}_\theta + \mu, 2\mu + 1, \rho\right), \quad (51)$$

and

$$\tilde{\Delta}_{\infty}^{-} = C_{\infty}^{-} (-\rho)^{\mu+1/4} e^{\rho/2} G\left(\frac{1}{2} + \tilde{k}_{\theta} + \mu, 2\mu + 1, -\rho\right), \quad (52)$$

where C_{∞}^{+} and C_{∞}^{-} are normalisation constants.

Now we utilise the relation:

$$\begin{aligned} G\left(\frac{1}{2} - \tilde{k}_{\theta} + \mu, 2\mu + 1, \rho\right) &= \frac{\Gamma(-2\mu)}{\Gamma\left(\frac{1}{2} - \tilde{k}_{\theta} - \mu\right)} F\left(\frac{1}{2} - \tilde{k}_{\theta} + \mu, 2\mu + 1, \rho\right) + \\ &+ \frac{\Gamma(2\mu)}{\Gamma\left(\frac{1}{2} - \tilde{k}_{\theta} + \mu\right)} \rho^{-2\mu} F\left(\frac{1}{2} - \tilde{k}_{\theta} - \mu, -2\mu + 1, \rho\right), \end{aligned} \quad (53)$$

where Γ is the Gamma function, and exploit the fact that the positive frequency mode $\tilde{\Delta}_{\infty}^{+}$ can be written in terms of the positive ($\tilde{\Delta}_0^{+}$) and negative ($\tilde{\Delta}_0^{-}$) frequency modes through the Bogouliubov transformation [35, 36, 37]:

$$\tilde{\Delta}_{\infty}^{+} = \hat{\alpha} \tilde{\Delta}_0^{+} + \hat{\beta} \tilde{\Delta}_0^{-}, \quad (54)$$

to find that $\hat{\alpha}$ and $\hat{\beta}$ are:

$$\hat{\alpha} = \frac{C_{\infty}^{+} \Gamma(-2\mu)}{C_0^{+} \Gamma\left(\frac{1}{2} - \tilde{k}_{\theta} - \mu\right)}, \quad \hat{\beta} = \frac{C_{\infty}^{+} \Gamma(2\mu)}{C_0^{+} \Gamma\left(\frac{1}{2} - \tilde{k}_{\theta} + \mu\right)} \exp(i\pi(\mu + 1/4)), \quad (55)$$

with

$$\frac{|\hat{\alpha}|^2}{|\hat{\beta}|^2} = \left| \frac{\Gamma\left(\frac{1}{2} - \tilde{k}_{\theta} + \mu\right)}{\Gamma\left(\frac{1}{2} - \tilde{k}_{\theta} - \mu\right)} \right|^2 \exp(2\pi\mu). \quad (56)$$

Using the following property of the Gamma function:

$$\left| \Gamma\left(\frac{1}{2} + i\rho\right) \right|^2 = \frac{\pi}{\cosh(\pi\rho)}, \quad (57)$$

and simplifying leads to:

$$\frac{|\hat{\alpha}|^2}{|\hat{\beta}|^2} = \frac{\cosh\left[\pi\left(-\tilde{k}_{\theta} + \mu\right)\right]}{\cosh\left[\pi\left(\tilde{k}_{\theta} + \mu\right)\right]} \exp(2\pi\mu). \quad (58)$$

The probability to create a single particle from vacuum is then:

$$P_k = \left(\frac{|\hat{\alpha}|^2}{|\hat{\beta}|^2} \right)^{-1} = \frac{\cosh\left[\pi\left(\tilde{k}_{\theta} + \mu\right)\right]}{\cosh\left[\pi\left(-\tilde{k}_{\theta} + \mu\right)\right]} \exp(-2\pi\mu). \quad (59)$$

Taking into account the fact that $\tilde{k}_\theta = \frac{m^2}{4eE} - \theta \frac{m^2}{4e}$, for small θ , we easily to obtain:

$$P_k = P_k(\theta = 0) + P_k^\theta, \quad (60)$$

where $P_k(\theta = 0)$ denotes the ordinary probability to create a single particle from vacuum in the presence of an electric field and has the expression:

$$P_k(\theta = 0) = \frac{\cosh[\pi(k + \mu)]}{\cosh[\pi(-k + \mu)]} \exp(-2\pi\mu), \quad k = \frac{m^2}{4eE}, \quad (61)$$

and P_k^θ is the generated non-commutative correction of order θ given by:

$$P_k^\theta = -\frac{\pi m^2}{4e} \theta P_k(\theta = 0) [\tanh \pi(k + \mu) + \tanh \pi(-k + \mu)]. \quad (62)$$

Next we calculate the non-commutative density of the created particles \hat{n} by the non-commutative curved space-time and electric field. For this we use eq. (54) so as to arrive at:

$$\hat{n} = |\hat{\beta}|^2. \quad (63)$$

Using the normalisation condition [37]:

$$|\hat{\alpha}|^2 - |\hat{\beta}|^2 = 1, \quad (64)$$

we finally arrive at the result for the non-commutative number density of created particles \hat{n} :

$$\hat{n} = \left(\frac{1 - P_k}{P_k} \right)^{-1} = \exp \left[\pi \left(\tilde{k}_\theta - \mu \right) \right] \frac{\cosh \left[\pi \left(\tilde{k}_\theta + \mu \right) \right]}{\sinh(2\pi\mu)}. \quad (65)$$

It is also very important to consider the weak and strong electric field limits and see the behavior of the number density and derive some of the related thermodynamical quantities.

3.1 The weak field approximation

In this limit, if we set:

$$\begin{aligned} \tilde{k}_\theta &= \frac{m^2}{4eE} (1 - \theta E) \\ &= \frac{m^2}{4eE} - \theta \frac{m^2}{4e}, \end{aligned} \quad (66)$$

such that:

$$\tilde{k}_\theta \rightarrow \infty, \quad (67)$$

it is easy to show that the probability P_k takes the form:

$$P_k = \exp \left[-\pi \left(\sqrt{k_\perp^2 - \frac{1}{4}} + \theta \frac{m^2}{4e} \right) \right]. \quad (68)$$

Then the number density \hat{n} is written up to the second order of θ as:

$$\hat{n} = \frac{1}{\exp \left[\pi \left(\sqrt{k_{\perp}^2 - \frac{1}{4}} + \theta \frac{m^2}{4e} \right) \right] - 1}. \quad (69)$$

This density is thermal and looks like a two-dimensional Bose-Einstein distribution with chemical potential $\mu^{\theta} = -\theta \frac{\pi m^2}{4e}$. To get the total non-commutative number of the created particles per a unit volume, we have to integrate the density \hat{n} over momentum space. Taking into account the fact that \hat{n} does not explicitly depend on k_z , the total non-commutative number \hat{N} reads:

$$\hat{N} = \frac{2}{(2\pi T)^2} \int \hat{n} k_{\perp} dk_{\perp} dk_z, \quad (70)$$

where T is the time for the external interaction and the integration over k_z is equivalent to the integration of the classical equation of motion: $\int dk = \int eEdt = eET$. Thus the total non-commutative number \hat{N} per a unit volume takes the form:

$$\hat{N} = \frac{2eET}{(2\pi T)^2} \left[\int \frac{k_{\perp} dk_{\perp}}{e^{\pi \left(\sqrt{k_{\perp}^2 - \frac{1}{4}} + \theta \frac{m^2}{4e} \right)} - 1} \right]. \quad (71)$$

Now since θ is small we have $\exp \left(\theta \frac{m^2}{4e} \right) \ll 1$. Consequently the total number \hat{N} in eq.(65), written up to the second order of θ , is given by the following relation:

$$\hat{N} \cong \frac{eE}{2\pi^4 T} \exp \left(-\pi \frac{m^2}{4e} \theta \right) \left(1 + \frac{1}{4} \exp \left(-\pi \frac{m^2}{4e} \theta \right) \right) + \mathcal{O}(\theta^2). \quad (72)$$

Notice that the particle creation mechanism is effectively isotropic in the presence of a constant electric field of the anisotropic Bianchi I universe of the non-commutative space-time. The non-commutative number density of created particles in eq.(66) takes a similar form in the Boltzmann limit in ordinary commutative space with a chemical potential $\mu^{\theta} = -\theta \frac{m^2}{4e}$. This result was expected due to the fact that the non-commutativity parameter is the smallest area in space that can be probed.

3.2 The Strong Field Approximation

In this limit if we set

$$\begin{aligned} \tilde{k}_{\theta} &= \frac{m^2}{4eE} (1 - \theta E) \\ &= k - \theta \frac{m^2}{4e}, \end{aligned} \quad (73)$$

such that:

$$k \rightarrow 0, \quad (74)$$

and by direct simplifications one may show that the number density \hat{n} takes the form:

$$\hat{n} = \frac{1}{\exp \left[\pi \left(\sqrt{k_{\perp}^2 - \frac{1}{4}} + \theta \frac{m^2}{2e} \tanh \pi \sqrt{k_{\perp}^2 - \frac{1}{4}} \right) \right] - 1}. \quad (75)$$

This result looks like a two dimensional Bose-Einstein distribution with a chemical potential μ^{θ} given by:

$$\mu^{\theta} = -\theta \frac{\pi m^2}{2e} \tanh \pi \sqrt{k_{\perp}^2 - \frac{1}{4}}. \quad (76)$$

Consequently the results shown in eqs. (69) and (75) indicate that the particle creation mechanism effectively isotropizes in the non-commutative space-time of an anisotropic Bianchi I universe, where the number density of created particles in the absence of the electric field corresponds to a thermal distribution. Here the non-commutativity parameter plays the role of electric field. In the limit $\theta \rightarrow 0$ equations (69) and (75) reduce to:

$$\frac{1}{\exp \left[\pi \left(\sqrt{k_{\perp}^2 - \frac{1}{4}} \right) \right] - 1},$$

which is the same as the Bose-Einstein distribution of particles (see [38]).

4 Conclusions

In this work we started with the effect of the non-commutativity on the creation of scalar particles from vacuum in the space-time anisotropic Bianchi I universe. By using the Seiberg-Witten maps and the Moyal product up to first order in the non-commutativity parameter θ , we generalised the deformed Klein Gordon equation. After solving this equation the density of created particles are calculated by applying the Bogoliubov transformations and the quasi-classical limit for identifying the positive and negative frequency modes. We have seen that the non-commutative space-time introduces a new source of particle creation by considering the application of our findings into the Bianchi I universe. As a conclusion, the non-commutativity plays the same role as the electric field and chemical potential. It is worth mentioning that in the limit $\theta \rightarrow E^{-1}$ (i.e. the case of strong field) and at high energies our results coincide with those of ref.[33].

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